XII. *Measurements of Electrical Constants.*—No. II. *On the Specific Inductive Capacities of Certain Dielectrics.**—Part I.

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Received October 21,—Read December 12, 1878.

Contents.

Introductory.

In the autumn of 1876 it was suggested in the course of a conversation with Professor Clerk Maxwell that I should, under his direction, undertake a series of measurements of specific inductive capacities—particularly those of transparent dielectrics—with a view of testing Professor Maxwell's electro-magnetic theory of

* A paper of mine with the above title was communicated to the Royal Society by Professor J. Clerk Maxwell, F.R.S., on March 9, 1878. It was read on March 28, and an abstract of it was published in the 'Proceedings of the Royal Society,' vol. xxvii., p. 270.

In the course of the summer it was pointed out to me that, owing to a mistake in the formula of calculation, all the results were wrong.

I therefore requested permission to withdraw my paper, in order to recalculate the results.

The new values of K arrived at led me to make some determinations of refractive indices, and to re-write the theoretical deductions at the end of the paper.

I now beg, through Professor Maxwell, to present the paper in an amended form, with the hope that it may be found not entirely unworthy the attention of the Royal Society.

light, which requires that the square of the refractive index should be equal to the dielectric capacity, multiplied by the magnetic permeability.

By March, 1877, the method of work had been sketched out, and most of the instruments ordered.

The greater portion of the expense of the investigation has been met by grants from the Government Fund of £4,000.

The general outline of the method, which is, with the exception of the first idea of the 5-plate balance,* entirely due to Professor MAXWELL, is as follows :—

Metal plates, some connected with the electrometer and some with the source of electrification, are so placed that, when there is only air between them, the potentials of the two pairs of quadrants are equal and the needle remains at zero, however the strength of the electrification may vary.

On placing one of the dielectrics under examination between two of the plates a stronger action takes place between these two plates, and the needle is deflected. The plates are now moved farther apart by a screw so as to reduce the action until the needle returns to zero.

From the distance which the plates have to be separated, and the thickness of the dielectric, the specific inductive capacity can be calculated.

The great difficulty in all experiments in specific inductive capacity is to make the experiments quickly enough. If the electric stress is continued in the same direction for any appreciable time, a permanent strain is produced, and the apparent specific inductive capacity differs from the true one.

To overcome this difficulty Professor MAXWELL arranged that the electrification should be constantly reversed. It was found possible to reverse it 12,000 times in a second, and so to eliminate all phenomena of "residual charge."

How, in spite of the constant reversal, it was arranged that the deflection, when the balance was not established, should be steady, and depend only on the position of the plates, and not on the direction of the electrification, will be seen later on in the paper.

Description of instruments employed.

For producing a rapidly reversed electrification of high potential, a coil, wheel-break, secondary reversing engine, and batteries were used.

The coil was one of APPS' induction coils, on which is wound 22 miles of secondary wire, and which, with a suitable battery and break, is capable of giving a 17-inch spark in air.

The wheel-break consists of a small magnetic engine the fly-wheel of which is of brass, and about 2½ inches in diameter. In the circumference of the wheel 60 slots are cut and filled with ebonite. A light spring faced with platinum presses on the edge of the wheel with a force regulated by a capstan screw.

* Due, I believe, to Sir WILLIAM THOMSON.

A current passing from the spring to the wheel would thus be made 60 times, and broken 60 times, in each revolution of the engine.

The engine, which is by APPS, is of particularly good construction for high speed work. Two horse-shoe electro-magnets are placed with their poles facing each other. One is fixed and the other revolves round an axis which is parallel to the cores and half way between them. The current in the fixed magnet has a constant direction, that in the revolving magnet is reversed every half revolution. The commutator is so arranged that the force between poles which are approaching each other is always attractive, and that between receding poles repulsive. Two screws regulate the pressure of the commutator springs.

With a little care as to the adjustment, it was found possible to drive this engine at a speed of 100 revolutions a second.

The secondary reversing engine consists of a square ebonite plate, near the corners of which are four holes filled with mercury. Upon it is supported an oscillating frame, which by dipping into the right and left hand pair of cups alternately, reverses at each oscillation any current passing through the instrument. The oscillations of the frame were produced by a rod going from it to a crank in the axis of a magnetic engine. The engine was similar to that of the wheel-break, except that as it was not driven at so high a speed, the screw adjustments of the springs were dispensed with.

To regulate the speed, a friction-brake was used, consisting of a loop of silk which passed round a pulley on the axis, and was connected by an india-rubber band to a cord. This cord could be tightened by turning a handle round which it was wound.

About 30 reversals per second was a sufficient speed for the purpose for which the engine was employed, and about the maximum which could be used without splashing the mercury from the cups.

The batteries.—Ten small LECLANCHÉ cells, in series, worked the coil, four pint GROVE's the wheel-break, and two pint GROVE's the reversing engine.

When the wheel-break was driven at full speed, the coil gave a spark about $\frac{1}{25}$ inch in length, the current being reversed 120 times in each revolution, or 12,000 times per second.

Comparison of make and break currents.

To test the equality of the make and break currents, the discharge was passed through a small vacuum tube. When the wheel-break was turned slowly the usual difference in the illuminations of the ends of the tube was observed, which difference was reversed on reversing the primary current of the coil. On working the break at high speed this difference disappeared, and no effect was produced by reversing the primary.

This shows that at high speed the sums of the make and break currents in opposite directions were approximately equal.

The secondary reversing engine was, however, used to guard against any difference in the above sums which might be produced by difference of leakage, &c., by reversing the secondary current about 30 times per second.

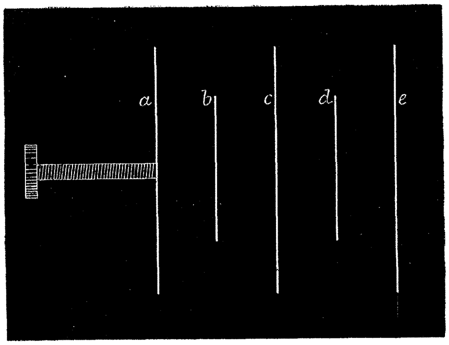
Description of instruments (continued).

The 5-plate induction balance.—This instrument was invented some years ago by Sir WILLIAM THOMSON.

Only a rough lecture-model was constructed at the time of the invention. The present instrument is the first which has been made for purposes of measurement. The working drawings were made by me from sketches and verbal explanations supplied by Professor MAXWELL.

The instrument was constructed with great skill by Mr. KIESER, of the firm of ELLIOTT Brothers.

Fig. 1.



It consists of three circular metal plates, *a c e*, each 6 inches diameter, and two, *b d*, each 4 inches diameter, arranged as shown in fig. 1. There is a space of rather more than an inch between each plate and the one next it. The plate *a* is movable parallel to itself, so that it can be placed either in contact with *b* or nearly three inches from it. The other four plates are fixed. The plates *b c d e* are supported from above by steel rods. The lower end of each rod is screwed into the upper edge of a plate, the upper into an ebonite plug fixed into a small triangular horizontal brass plate, at the corners of which are levelling screws. The screws rest on a flat brass stage, a slit in which allows the rod to pass through. As there is not room for all four triangles close together there are two stages one above the other.

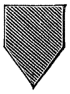
The plates *c e* hang from the lower stage, and *b d*, which are furnished with longer rods, from the upper one. The triangles when adjusted are clamped by screws which for *c e* are fixed in the second stage, while for *b d* the clamps are carried on a third stage made especially to hold them. Four stout brass pillars support the three stages. The feet of these pillars are screwed into a large brass plate let into the wooden base of the instrument. On this plate also stand the pillars carrying plate *a*. Plate *a* is fixed to the end of a brass rod of section  from which it is insulated by a block of ebonite (E, fig. 3). This slides on two pillars (A A, figs. 2 and 3), which it touches only on its inclined surfaces (fig. 2). It is pressed downward by stout springs (*α α*, figs. 2 and 3). It is moved backwards and forwards by a screw B, pressed by a spring against a hardened steel plate at C (fig. 3). The screw inside D turns in two collars, one fixed to D and the other only kept from revolving, and forced away from the first by a stout spiral spring. This prevents what is called "back lash," *i.e.*, it ensures that the longitudinal motion shall be reversed at the same time as the motion of the screw.

Fig. 2.

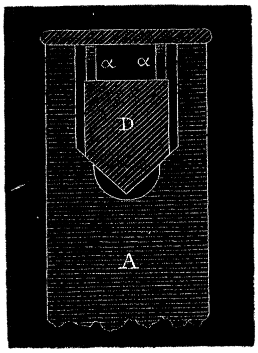
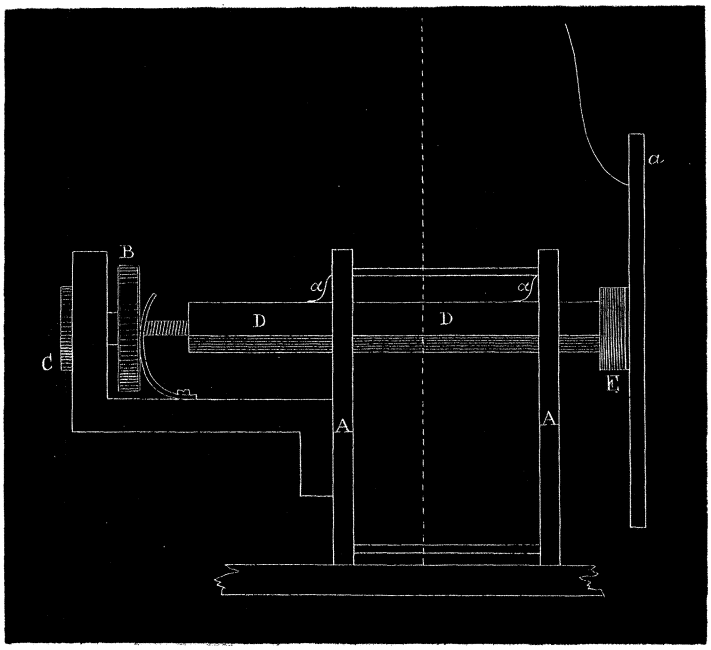


Fig. 3.



A scale divided to $\frac{1}{50}$ inch is engraved upon D, and a vernier fixed to one of the uprights reads the position of the plate to $\frac{1}{1000}$ inch. The scale is read by a microscope, fixed some three inches distant on the case of the instrument.

The five plates are enclosed in a glass case, like a balance case, about 15 inches long. It passes below the stages, and holes in the top glass admit the four steel rods. A hole in the side admits the rod D, so that a is inside the case, while the screw-head is outside. The dotted line in fig. 3 shows the position of the glass side of the case.

The five plates are placed close to one end of the case (see fig. 5, *Induction Balance*). On the base, inside the case, are slides, in which stages for carrying the dielectrics move. The stages can be moved by rods projecting beyond the case. Thus a dielectric under examination can be placed between two of the plates, or removed from them, without opening the glass case.

On the upper fixed stages are ebonite pillars with double-binding screws on them; to the lower nut of each is attached a flexible spiral wire, leading to one of the plates by way of the steel rod. By means of the upper nuts the plates can be connected to other instruments.

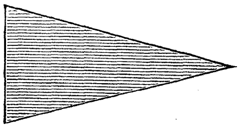
The mechanical slide.—One of the sliding stages—viz.: that used to place a dielectric between a and b*—has "mechanical motions." In addition to the rod by which it is drawn in and out, there are three other parallel rods, with milled heads. Turning, one gives a lateral motion—viz.: moves any dielectric placed on the slide nearer a or b. By turning the second the dielectric plate can be placed either exactly vertical or inclined a little in either direction; and turning the third gives the dielectric a small angular motion round a vertical axis.

The THOMSON *electrometer* is a quadrant of the simple form by ELLIOTT; one of WHITE's which I have was found to be unsuitable for use with the reversing gear. When, as in this case, the instrument is only used as an electroscope, the superior sensitiveness of ELLIOTT's pattern gives it great advantages.

The callipers were a pair with especially long jaws, made for use in adjusting the plates of the balance. When laid upon a bracket fixed to the outside of the balance case, the jaws projected right in between the plates. This bracket could be inclined either up or down, for measuring at the upper or lower parts of the plates. The "outsides" scale was used in the ordinary way for measuring the thickness of the dielectric plates.

Determination of speed of wheel-break.

The fly-wheel of the engine had five spokes. A piece of card of this shape



being held with its pointed end against the spokes, was struck five times in each revolution, and gave the note B=  which corresponds to 495 vibrations per second.† Dividing this by 5, we find that the wheel revolves 99 times

* In practice, the dielectrics are only put between a and b.

† 'Sound and Music,' SEDLEY TAYLOR, p. 200.

per second—say 100, as the card would check the speed slightly. This determination was made with a battery which had been charged 24 hours, and had done a good deal of work.

Estimation of electromotive force of secondary current.

The spark given by the coil with the wheel-break and LECLANCHÉ battery is about ·04 inch. Let N be the number of cells this corresponds to. Now Messrs. DE LA RUE and MÜLLER have* enunciated the following formula for the spark produced by any number of their cells:—

$$\text{Length of spark} = \frac{\cdot 0033 N^2}{600^2}$$

$$\text{Here length} = \cdot 04.$$

Hence we have

$$N^2 = \cdot 04 \frac{600^2}{\cdot 0033} = 4{,}300{,}000$$

$$N = \text{about } 2050.$$

The electromotive force is then equal to that of about 2050 cells of a chloride of silver battery.

This determination was made with the coil disconnected from the induction balance. When it is connected to it the spark obtainable is much shorter, owing, possibly, to the leakage, or more probably to the capacity of the induction balance and electrometer, a certain proportion of the electricity being required to charge these at every make and break. I had no means of accurately measuring the lengths of very short sparks.

Connections and general method of working.

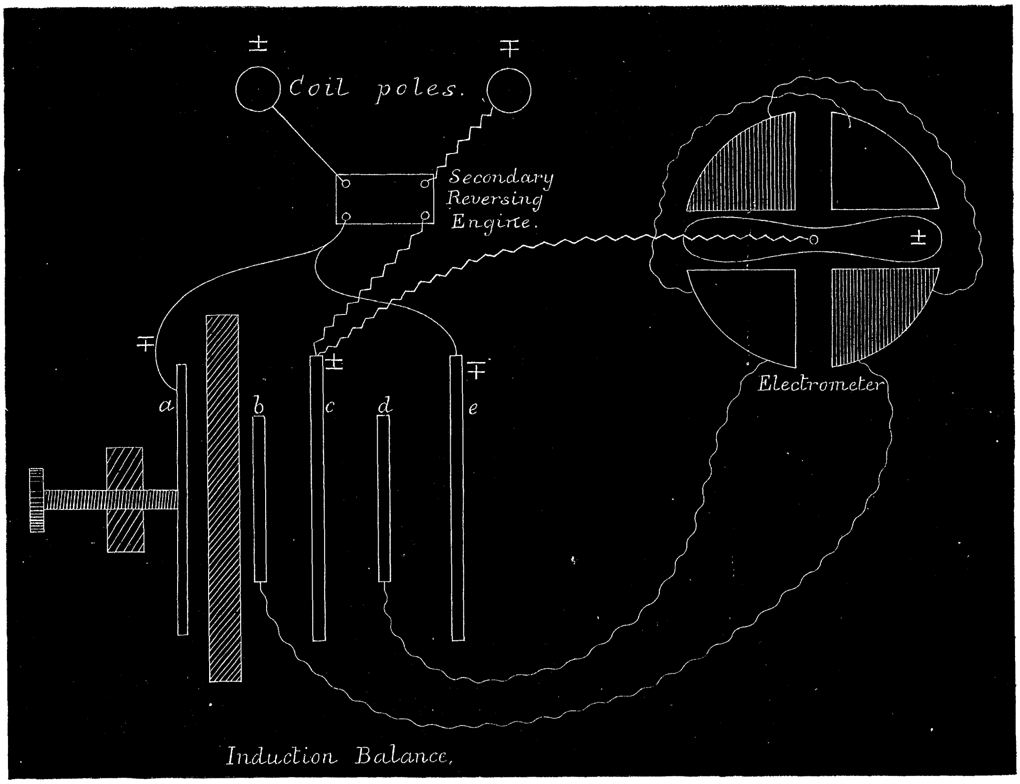
The connections are shown in figs. 4 and 5. The wheel-break and LECLANCHÉ battery were connected to the coil primary in the usual way. The secondary current after passing the secondary reversing engine came to the induction balance.

One of the wires carrying the secondary current was connected to the centre plate c (fig. 4), the other to the two outer ones, a and e; the smaller plates b and d were connected to the quadrants of the electrometer respectively. Now, for a moment, let us suppose the electrification to be produced by a battery acting constantly in the same direction, and the needle of the electrometer to have a permanent charge; then when the plates are all placed symetrically and with only air between them, however strong the battery may be, the spot of light will remain at zero. This may be seen by noting in fig. 4 the actions of the large plates on the small ones respectively. If, however, a is moved away so as to make the distance $a\ b$ greater than $d\ e$, the needle will be deflected in one direction. If instead of moving a away we had placed a dielectric plate in the space $a\ b$, there would have been a greater action from a to b, and the

* 'Proc. Roy. Soc.,' 1875–76, p. 170.

deflection would have been in the opposite direction. It is clear that if we insert a dielectric, and also move *a* away from *b*, we can, by properly adjusting the motion, keep the electrometer at zero.

Fig. 4.

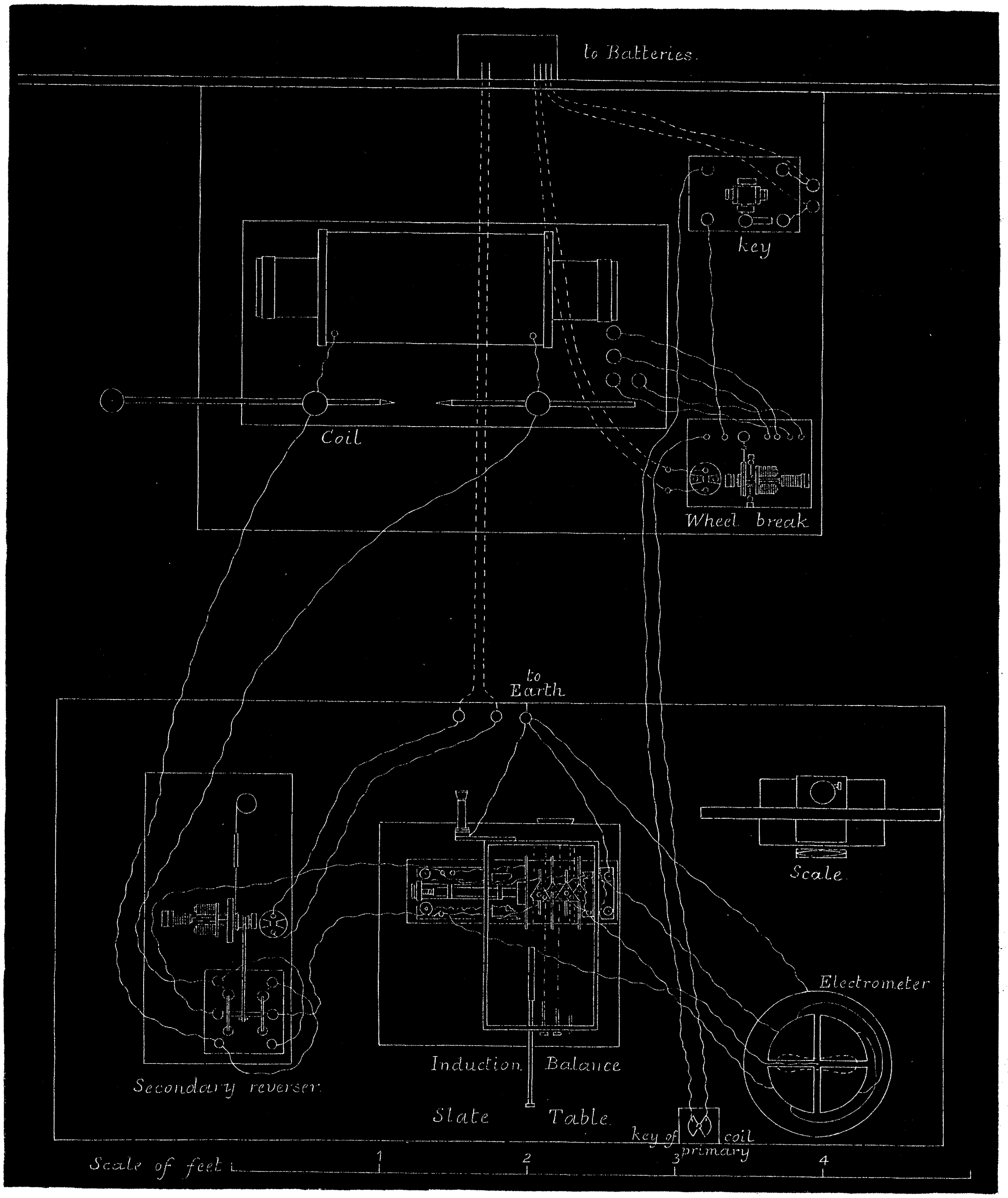


The amount of motion of *a* required in each case, will depend on the thickness of the dielectric, *and on its specific inductive capacity.*

Let us suppose the balance not perfectly established, and a deflection, say, to the right—now reverse the poles (we are still supposed to be working with a battery); the deflection will be to the left, and with the electrifications which are actually used, and which are reversed 12,000 times in each second, no disturbance of equilibrium would produce any deflection if the needle were charged in the ordinary way.

To escape from this difficulty Professor MAXWELL devised the following arrangement:—The needle, instead of being permanently charged, was connected with the plate *c*, and every time the electrification of the plates was reversed, that of the needle was reversed also. As the sign of the force between two electrified bodies depends only on whether the electrifications are like or unlike, and not on whether they are positive or negative, this arrangement caused the force in the needle to depend only on the numerical difference between the potentials of the quadrants, and not on the sign of the electrification.

Fig. 5.


to Batteries.
key
Coil
Wheel break
to Earth
Scale
Electrometer
Induction Balance
Slate Table
Secondary reverser
key of primary coil
Scale of feet
1
2
3
4

Thus the electrifications, of the needle, the five plates, the dielectric, and the four quadrants were all reversed 12,000 times in each second, and yet the light spot was found to be absolutely steady, and completely under the control of the screw of *a*.

A motion of *a* of ·001 inch usually caused a deflection of from 1 to 2 millims. on the scale. I so arranged the connections that, to bring the light spot to zero, *a* had to be moved so as to follow the deflection. This was a very convenient way of remembering in which direction to turn the screw.

The actual determinations were made as follows:—The dielectric was removed and the slide on which it stood being pushed in under the plates, the position of *a* was determined which brought the light spot to zero with only air in the balance. The slide was then drawn out, the dielectric placed upon it, and it was than pushed in so as to place the dielectric between *a* and *b*. The dielectric plates were all 7 inches high and 7 inches or 8 inches long, and varied from $\frac{1}{4}$ inch to 1 inch in thickness. A second reading was taken when the dielectric was in position.

Before each set of measurements, preliminary experiments were made by means of which *a* was placed very nearly in its right position in each case before the primary circuit was closed. This prevented large deflections of the needle, which were found injurious, as after a very large deflection the light spot seldom returned exactly to the same zero.

Before commencing a measurement the engines were started: the observer standing where he could see the scale, was able with his right hand to make and break the coil primary circuit, and with his left to work the screw of *a* (see fig. 5). As soon as equilibrium was established, contact was broken, the engines stopped, and the reading of *a* taken with the microscope.

The formula used for calculating the results assumes that the lines of force through the dielectric are straight and perpendicular to its sides. Now the lines of force are very nearly straight over most of the space *a b*, but not quite straight close to either *a* or *b*. When the lines of force through the dielectric are perpendicular to its sides they will be at their shortest in it, and it will produce its minimum effect. By means of the mechanical slide the dielectric was in each case adjusted to give its minimum effect. The field of force was, however, found to be very nearly uniform except close to the plates. A cardboard box covered with tinfoil, and connected to earth, protected the second stage which carried the electrometer connections.

All wires leading from the quadrants to the balance were protected by wide cardboard* tubes covered with tinfoil connected to earth. The metal portions of the cases of balance and electrometer were also connected to earth.

* May, 1879.—Brass tubes and a brass box over the upper stage have since been substituted with advantage.

The formula of calculation.

Let the reading of a when there is only air in the balance be a_1, and that when the dielectric is inserted a_2.

A dielectric plate of thickness b being inserted displaces a thickness b of air. A dielectric plate of thickness b and specific inductive capacity K acts like a plate of air of thickness $\frac{b}{K}$.

Therefore the effect of inserting the plate has been the same as if we had decreased the distance $a\,b$ by a quantity $b-\frac{b}{K}$.

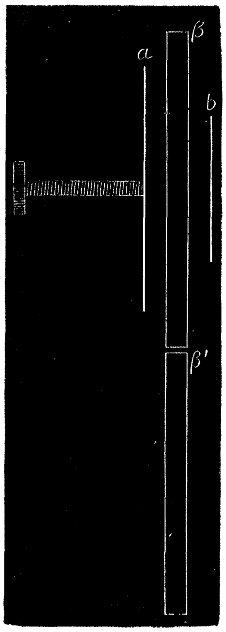
But when we have again brought the electrometer to zero by the screw we have increased $a\,b$ by a quantity a_2-a_1, and as this increase exactly compensates the apparent decrease produced by the dielectric, we must have

$$b-\frac{b}{K}=a_2-a_2, \qquad \text{or} \qquad K=\frac{b}{b-(a_2-a_1)}$$

and this formula was used to calculate the results of the following experiments.

The experiments on ebonite show that the results given by the formula agree with each other over a very wide range of values of b.

Fig. 6.



This formula assumes that those lines of force between a and b which pass through the dielectric are approximately straight lines, also that all the lines pass through it, and that no portion, or at least no sensible portion, of them pass round its edge. To test the truth of this assumption the following experiment was made:—A plate of ebonite, β, fig. 6, being placed in the usual way between a and b, a was adjusted so as to bring the light spot to zero. The circuit being kept closed, a second similar piece of ebonite was placed at β'. Now, if any of the lines of force had curved round the edge of β, the introduction of β' would have caused a deflection in the direction showing an increase of specific inductive capacity in $a\ b$. No such deflection however took place, which shows that no lines of force strong enough to affect the electrometer pass round the edge of a 7-inch dielectric.

The experiments.

After about five months spent in preliminary experiments and in perfecting the adjustments of the various instruments, the following measurements were made :—

Optical glass.—Four beautiful slabs, each seven inches square, of perfectly homogeneous glass, were cast in Birmingham by Messrs. CHANCE, and polished in London under the directions of Mr. KIESER.

The glasses were—

Double extra-dense flint.
Extra-dense flint.
Light flint.
Hard crown.

Small prisms of each glass were also prepared in order that the refractive indices might be compared with the specific inductive capacities.*

The following are the details of the experiments and values of K deduced from them :—†

Optical glass.

DOUBLE extra-dense flint.

$b=\cdot 847.$

a_2	(1) 1·613	(3) 1·615	(5) 1·610
a_1	(2) 1·034	(4) 1·034	1·032
a_2-a_1	·579	·581	·578

Mean ·5793.

K=3·164.

* See p. 440.

† a_1, a_2, and b are all given in inches.

EXTRA-dense flint.

$b = \cdot 717$.

a_2	(2) 1·514	(4) 1·515	(6) 1·511
a_1	(1) 1·032	(3) 1·031	(5) 1·030
$a_2 - a_1$	·482	·484	·481

Mean ·48223.

K=3·054.

LIGHT flint.

$b = \cdot 699$.

a_2	(1) 1·503	(3) 1·498	(5) 1·495	(7) 1·500
a_1	(2) 1·035	(4) 1·034	(6) 1·032	(8) 1·032
$a_2 - a_1$	·468	·464	·467	·468

Mean ·46675.

K=3·013.

HARD crown.

$b = \cdot 864$.

a_2	(2) 1·621	(4) 1·617	(6) 1·616	(8) 1·619
a_1	(1) 1·032	(3) 1·032	(5) 1·031	(7) 1·034
$a_2 - a_1$	·589	·585	·585	·585

Mean ·586.

K=3·108.

Ordinary plate glass.

$b = \cdot 996$.

a_2	1·733	1·734	1·731
a_1	1·043	1·042	1·042
$a_2 - a_1$	·690	·692	·689

Mean ·6903.

K=3·258.

ANOTHER piece of the same.

$b=1{\cdot}003$.

a_2	1·739	1·737	1·730
a_1	1·044	1·043	1·042
a_2-a_1	·695	·694	·688

Mean ·6923.
K=3·228.

Ebonite.

Four slabs of different thickness.

No. 1.

$b=\cdot754$ (mean).

a_2	(1) 1·460	(3) 1·453	(5) 1·453	(7) 1·452
a_1	(2) 1·031	(4) 1·032	(6) 1·036	(8) 1·032
a_2-a_1	·429	·421	·417	·420

Mean ·42175.
K=2·2697.

No. 2.

$b=\cdot509$ (mean).

a_2	(2) 1·313	(4) 1·318	(6) 1·315
a_1	(1) 1·032	(3) 1·033	(5) 1·033
a_2-a_1	·281	·285	·282

Mean ·2826.
K=2·2482.

No. 3.*

$b=\cdot516$.

a_2	(1) 1·325	(3) 1·323	(5) 1·321
a_1	(2) 1·030	(4) 1·030	1·031
a_2-a_1	·295	·293	·290

Mean ·2926.
K=2·3097.

* Slabs Nos. 3 and 4 were much warmer than Nos. 1 and 2 when experimented on. But on the experiment with No. 4 being repeated after it was cool no difference was perceptible.

No. 4.

$b = \cdot 264$ (mean).

a_2	1·184	1·181	1·180
a_1	1·031	1·032	1·033
$a_2 - a_1$	·153	·149	·147

Mean ·1496.
K=2·3077.

SUMMARY of experiments on ebonite.

Slab.	Thickness.	Spec. ind. capacity.
	inch.	
No. 1	·754	2·2697
,, 2	·509	2·2482
,, 3	·516	2·3097
,, 4	·264	2·3077
Mean		2·284

Extreme difference from the mean= −1·5 per cent.

Vulcanized india-rubber.

$b = \cdot 789$.

a_2	(2) 1·506	(4) 1·523	(6) 1·505	1·505
a_1	(1) 1·035	(3) 1·035	(5) 1·041	1·035
$a_2 - a_1$	·471	·488	·464	·470

Mean ·47325.
K=2·497.

Plain black india-rubber.

$b \begin{cases} \cdot 992 \\ 1\cdot 006 \end{cases}$ mean ·999

a_2	1·586	1·590	1·590
a_1	1·042	1·038	1·037
	·544	·552	·553

Mean ·5496
K=2·220.

Best quality gutta percha.

$$b\left\{\begin{array}{c}\cdot786\\ \cdot789\\ \cdot787\\ \cdot789\end{array}\right\}\text{mean }\cdot788$$

a_2	(2) 1·517	1·515	1·515
a_1	(1) 1·048	1·048	1·047
a_2-a_1	·469	·467	·468

Mean ·468.
K=2·462.

CHATTERTON'S *compound.*

$$b\left\{\begin{array}{c}\left.\begin{array}{c}\cdot842\\ \cdot834\end{array}\right\}\cdot834^{*}\\ \left.\begin{array}{c}\cdot826\\ \cdot830\end{array}\right\}\cdot832\end{array}\right\}\text{mean }\cdot833$$

a_2	1·555	1·555	1·555
a_1	1·048	1·049	1·048
a_2-a_1	·507	·506	·507

Mean ·5066.
K=2·547.

Sulphur.

This plate was not a very good one, and was cracked across one corner.†

$$b\left\{\begin{array}{c}\cdot745\\ \cdot751\end{array}\right\}\text{mean }\cdot748.$$

a_2	1·496	1·492	1·507
a_1	1·035	1·037	1·038
a_2-a_1	·461	·455	·469

Mean ·4583.
K=2·58.

* The four measures being taken at the four sides in order, the agreement of these two means shows that the slope is uniform.

† It was prepared by casting in a mould with glass sides. It was semi-transparent for some hours after it had set, but became opaque before the experiments were made.

Shellac.

b measured from centre of each of the four sides.

$$\left.\begin{array}{l}\cdot 791\\ \cdot 779\\ \cdot 743\\ \cdot 772\end{array}\right\}\text{mean }\cdot 769$$

a_2	(1)	1·536	1·536	1·536
a_1	(2)	1·047	1·047	1·046
a_2-a_1		·489	·489	·490

Mean ·4893.
K=2·74.

Solid paraffin (Messrs. CLARK and MUIRHEAD'S).

Six slabs cut in planing machine from larger blocks.

No. 1.

b=·730.

a_2	(1)	1·382	1·378	1·379	1·379
a_1	(2)	1·030	1·029	1·027	1·028
a_2-a_1		·352	·349	·352	·351

Mean ·351.
K=1·9261.

No. 2.

b=·750.

a_2	(2)	1·392	1·392	1·393
a_1	(1)	1·037	1·034	1·035
a_2-a_1		·355	·358	·358

Mean ·357.
K=1·9084.

No. 3.

$b = \cdot748$.

a_2	(1)	1·397	1·398	1·397
a_1	(2)	1·035	1·035	1·037
$a_2 - a_1$		·362	·363	·360

Mean ·3616.

K=1·9358.

No. 4.

$b = \cdot782$.

a_2	(2)	1·425	1·418	1·421
a_1	(1)	1·037	1·035	1·037
$a_2 - a_1$		·388	·383	·384

Mean ·385.

K=1·9697.

No. 5.

$b = \cdot755$.

a_2	(1)	1·402	1·402	1·404
a_1	(2)	1·037	1·036	1·037
$a_2 - a_1$		·365	·366	·367

Mean ·366.

K=1·9408.

No. 6.

$b = \cdot754$.

a_2	(2)	1·411	1·411	1·411
a_1	(1)	1·037	1·034	1·034
$a_2 - a_1$		·374	·377	·377

Mean ·376.

K=1·9947.

Correction for cavities in paraffin.—All the plates contain cavities more or less numerous. The following approximate method of allowing for the effect of these cavities was employed:—

In the calculations we substitute for the plate of thickness b, an imaginary plate whose thickness is that which the plate would have had if it had been of the same length and breadth, contained the same quantity of paraffin, and had no cavities. Let us call this thickness b', and we shall have

$$\frac{b}{b'}=\frac{\text{density of solid paraffin}}{\text{density of plate}}$$

The above will not be a complete correction, owing to the unequal distribution of the cavities, but when the formula is applied to each of the six determinations, their mean should not be far from the truth.

To determine the density of the plates, two methods were used. One method was to carefully plane their edges, and then weigh and measure each plate, and so determine the weight of a cubic inch of each of them. The other was to weigh the plates in water with a lead sinker. The objection to the second method was, that some of the plates had cavities open to the edges into which the water could run. On trying both methods, however, the uncertainty introduced by this was found to be less than that caused by errors of measuring.

The specific gravity of a small flat piece of paraffin, free from cavities, was determined in the usual way by means of a fine balance of ŒRTLING'S.

	grammes.
Weight of paraffin in air	2·164
Weight of paraffin and lead sinker in water at 11° C.	2·090
Weight of lead sinker in water	2·290
Thence weight of paraffin in water	— ·200
Weight of suspending hair	·005
Specific gravity of paraffin at 11° C.	=·9109

Dr. MUIRHEAD kindly made another determination for me, and found specific gravity =·912, but he did not state the temperature.

To determine the density of the paraffin plates a much larger balance was used, which was provided only with English weights. One end of the beam projected over the edge of the table, and the plates were slung horizontally in a double stirrup of fine iron wire, which was attached to the balance by a wire about three feet long. The weight of the wire and stirrup was compensated in air. The difference in its weight caused by immersing the lower portion of it in water was neglected. The plate hung in a large foot-pan full of water placed on the floor. The lead sinker was simply laid on the plate. When the balance was loaded with 18 ounces, it turned with about 2 grains. Weight of lead sinker in water, 1 lb. 4 oz. 4 dr. 1 scr. 2 gr.

WEIGHTS of plates in water equal weights of plates with sinker, minus weight of sinker in water.

Temperature of water 11° C.

Plate.	Weight in air in grains.	Weight in water in grains.	Specific gravity at 11° C.
No. 1	9600	−1330	·8783
,, 2	9282	−1330	·8771
,, 3	9904	−1160	·8951
,, 4	10,797	−1322	·8909
,, 5	10,027	−1201	·8933
,, 6	10,585	−1149	·9021

Let us write

$$\phi=\frac{\text{Sp. gr. of plate}}{\text{Sp. gr. of paraffin}}=\frac{\text{Sp. gr. of plate}}{\cdot 9109}$$

We shall then have

$$b'=b\phi.$$

Let us call K′ the corrected value of K, that is, the value of K calculated from b' instead of from b, and we shall have from the experiments, and the above determinations the following:—

SUMMARY of experiments on paraffin.

Plate.	ϕ.	b.	b'.	K.	K′.
No. 1	·9642	·730	·7038	1·9261	1·9940
,, 2	·9628	·750	·7221	1·9084	1·9784
,, 3	·9826	·748	·7249	1·9358	1·9969
,, 4	·9780	·782	·7648	1·9697	2·0126
,, 5	·9868	·755	·7450	1·9408	1·9654
,, 6	·9903	·754	·7467	1·9947	2·0143

$$\text{Mean value of}\begin{cases}\text{K}=1\cdot 9459\\ \text{K}'=1\cdot 9936^*\end{cases}$$

Among the corrected values the extreme difference from mean is −1·4 per cent.

* N.B.—In future this number will be called K.

Among the uncorrected values the extreme difference from the mean is $+2{\cdot}5$ per cent., which shows that though the correction is a rough one it gives numbers more nearly true than the uncorrected experiments.

The melting point of the paraffin was 68° C.

Bisulphide of carbon.—The liquid was contained in a glass trough. In this case a_1 was the reading when the empty trough was placed in $a\ b$; a_2 the reading when it was full. The trough was 7 inches wide and 9 inches high; b is its internal thickness.

We have from direct measurement $b = {\cdot}637$. The external thickness was ·779. The thicknesses of the sides ·076 and ·066, respectively, and $\cdot 779-(\cdot 076+\cdot 066)=\cdot 637$; showing that the inside and outside scales of the callipers agree very well.

The experiments had to be made somewhat hastily owing to leakage of the bisulphide.

$$b = \cdot 637$$

a_1	1·192		1·189		1·192
a_2		1·471		1·467	
a_1	1·176		1·173		

Mean $a_2 = 1{\cdot}469$

Mean $a_1 = 1{\cdot}184$

$a_2 - a_1$ ·285

$$K = 1{\cdot}81.$$

The following table gives a general summary of the results of the experiments, and compares the values of K obtained with those given by various former experimenters:*—

* Some of the numbers given in the third column are from 'Units and Physical Constants,' by Professor EVERETT, p. 134 (Macmillan, 1879). Where no reference is given, the number is from a list sent to me by Professor MAXWELL.

Name of dielectric.	K as determined by present experiments.	K as determined by experimenters mentioned in next column.	Previous experimenters.	References.
Glass—				
Double extra-dense flint	3·164*	10·1	HOPKINSON	'Proc. Roy. Soc.,' 1877, p. 298, vol. xxvii.
Extra-dense flint	3·054.*			
Dense flint		7·4	HOPKINSON	,, ,,
Light flint	3·013*	6·85	HOPKINSON	,, ,,
Hard crown	3·108.*			
Common plate, two specimens—				
No. 1	3·258 } mean 3·243	6·10	WÜLLNER	'Sitzb. Königl. bayer. Akad.,' March, 1877.
,, 2	3·228			
Halbweisses glass		2·96 } mean 3·31	SCHILLER.	
		3·66		
White murren		5·83	SCHILLER	Pogg. Ann., vol. clii., p. 535, 1874.
Common flint		Greater than 1·76	FARADAY	'Exp. Res.'
Ebonite, four slabs—				
No. 1	2·2697 } mean 2·284	2·76	SIEMENS.	
,, 2	2·2482	3·15	BOLTZMANN	'Weiner Sitzb.,' Jan. 9, 1873.
,, 3	2·3097	2·21	SCHILLER.	
,, 4	2·3077	2·56	WÜLLNER	'Sitzb. Königl. bayer. Akad.,' March, 1877.
Best quality gutta percha	2·462.			
CHATTERTON'S compound	2·547.			
India-rubber—				
Black	2·220	2·12 to 2·34	SCHILLER	Pogg. Ann., vol. clii., p. 535, 1874.
Grey vulcanized	2·497	2·69 to 2·94	SCHILLER	
Solid paraffin, specific gravity at 11° C. =·9109, melting point 68° C. Six slabs cut in planing machine; results corrected for cavities—		2·32	BOLTZMANN	CARL'S 'Repertorium,' vol. x., pp. 92, 165.
		1·96	WÜLLNER	'Sitzb. Königl. bayer. Akad.,' March, 1877.
No. 1	1·9940 } 1·9936	1·977	GIBSON and BARCLAY	Phil. Trans., 1871, p. 573.
,, 2	1·9784			
,, 3	1·9969			
,, 4	2·0126	Specific gravity at 11° C., calculated from data in Messrs. GIBSON and BARCLAY'S paper, =·9080. Multiplying Messrs. GIBSON and BARCLAY'S value of K by ratio of specific gravities, we have 1·9833 for my paraffin by GIBSON and BARCLAY'S method.		
,, 5	1·9654			
,, 6	2·0143			
Paraffin, clear, quickly cooled		1·68	SCHILLER.	
Slow cooled, white		1·81 } 1·85	SCHILLER.	
		1·89		
Shellac	2·74	2·00	FARADAY	'Exp. Res.,' 1270.
		2·95 to 3·73	WÜLLNER	'Sitzb. Königl. bayer. Akad.,' March, 1877.
Sulphur	2·58	2·88 to 3·21	WÜLLNER	,, ,,
		2·24	FARADAY	'Exp. Res.,' 1266.
		3·84	BOLTZMANN	CARL'S 'Repertorium,' vol. x., pp. 92, 165.
Bisulphide of carbon	1·81.			

* Note added September 25, 1879.—At the time (Christmas, 1877) when these experiments were made the glasses were newly cast. On repeating the experiments in August, 1879, it was found that the specific inductive capacities of all the optical glasses had considerably increased in the course of eighteen months, the new values being: D.E.D.F., 3·838; E.D.F., 3·621; L.F., 3·443; H.C., 3·310. For full details of these experiments see 'Report of the British Association,' 1879, p. 250.

It has been suggested to me that I should give some evidence that—

(1.) The insulation of the apparatus was perfect.
(2.) That the plates were good insulators.
(3.) That their surfaces were not damp.

With regard to the first point, an examination of the method will show that an absolutely perfect insulation is not necessary.

As long as no electricity leaked to the small plates *b* and *d*, a leakage from the large plates *a c e* would only produce the same effect as a diminution of the electromotive force of the coil current. It would diminish the sensitiveness of the instrument, but would not affect the results obtained by it.

To prevent the electricity which leaked from the large plates leaking to, or acting by induction on, the small ones, the wires and supports belonging to the latter were completely protected by metal screens well connected to earth.

The efficiency of this protection was shown by the perfect steadiness of the light spot when the earth connection was good, and its perfect obedience to the screw of *a*.

The instant, however, that from any cause the earth connection became imperfect, the instrument became entirely unmanageable, and the light spot was in continual irregular and violent motion. I have, for the continuation of the investigation, lately introduced some further improvements, both in the insulation and in the arrangement of the metal screens.

No error is to be anticipated from defective insulation, as with the apparatus in a leaky state, instead of incorrect results being obtained, measurement becomes impossible.

With regard to any possibility of leakage occurring to one of the small plates, I may add that, after a few months' experience of the balance, it would be impossible to mistake a deflection due to this cause for the regular deflection caused by induction.

As to the second point, the only proof that the plates are good insulators is that the substances of which they are composed, glass, paraffin, &c., are generally considered to be so.

With respect to the possible existence of a film of moisture, the following is the state of the case :—

Before putting any plate into the balance it was cleaned and rubbed dry with a wash-leather. It was then rapidly passed over the flame of a spirit lamp to discharge any electrification that it might have acquired from the friction.

The spirit lamp, of course, deposited a dew on the surface. This, however, was very slight, and could be seen to evaporate rapidly in the course of a few seconds. In one or two cases the plates were carefully dried by a fire, but no particular difference was noticed.

The exact agreement with each other of the experiments on those substances, of which I possessed more than one plate, such as paraffin, ebonite, and particularly common plate glass, shows, I think, that this film of moisture can never have produced any serious effect, as the experiments were made on many different days during which the temperature and dew-point varied very considerably.

The effect of the film of moisture, if any, would have been to decrease a_2-a_1. Now as each set of determinations took about 20 minutes, the film, if it existed, would have dried gradually during this time, and in the last experiments of a set, a_2-a_1 would always have been greater than in the earlier ones. An examination of the details of the experiments does not show that this was the case.*

The close agreements of the values of K for paraffin found by me and by Messrs. GIBSON and BARCLAY, who worked by an entirely different method, shows that there cannot be any serious defect either in their method or in mine.

Measurement of the refractive indices of the transparent dielectrics.

In order to see how far the results of these experiments bore out Professor MAXWELL'S electromagnetic theory of light, the square roots of the specific inductive capacities of the various transparent dielectrics were compared with the refractive indices given in works on physics. Wherever an approximate agreement was found the refractive index was carefully re-determined. It was not, however, thought necessary to make experiments on the refractive indices of the dielectrics when either there was an obvious and wide difference between μ and $\sqrt{K}$, or when, as in the case of paraffin, a previous determination, probably more trustworthy than any I could make, was available.

Optical glass.—In order to see whether Dr. HOPKINSON'S† determinations of the refractive indices of CHANCE'S optical glass could be compared with those obtained by me, the specific gravities of the specimens used by me were taken and compared with the specific gravities of those used by Dr. HOPKINSON.

The following were the results:—

The name given to the glass by Messrs. CHANCE.	Specific gravity of Dr. HOPKINSON'S specimens.	Specific gravity of GORDON'S specimens.
Double extra-dense flint	4·42162	4·421
Extra-dense flint	3·88947	3·884
Light flint	3·20609	3·198
Hard crown	2·48664	2·484

* September 25, 1879.—Some later experiments on the effect of wetting the plates have shown that no error could have been introduced accidentally by the presence of a film of moisture.—'Rep. Brit. Assoc., 1879, p. 250.

† 'Proc. Roy. Soc.,' 1877, p. 290.

Experimental determinations of μ.—A large 4-prism spectroscope by BROWNING was used.

It has a silver circle about $9\frac{1}{2}$ inches in diameter or nearly 30 inches in circumference.

The vernier reads to 10″.

The prisms were removed and replaced by one prism of the glass under examination.

μ was determined in the ordinary way by first measuring the angle of each prism and then mapping a spectrum observed through it at minimum deviation.

An attempt was made to observe the solar spectrum, but owing to the uncertainty of the weather in October it was abandoned, and a spark between magnesium poles in air was used as the source of light instead.

The spark was produced by means of my 17-inch coil working with its own vibrating break. A Leyden jar was connected to the secondary circuit in the usual way, and a convenient discharging frame caused the spark to take place in front of the slit.

The wires of the primary circuit were brought round to a plug-key fixed to the table close to the spectroscope. This enabled the observer to start and stop the coil without leaving his place.

To determine the refractive index for waves of infinite length, we proceed as follows :—

We have the general equation

$$\mu=A+\frac{B}{\lambda^2}$$

To determine A, a determination of the values of μ for two rays of different wave lengths λ and λ' are necessary and sufficient, for we have

$$\mu\lambda^2=A\lambda^2+B$$
$$\mu'\lambda'^2=A\lambda'^2+B$$

Subtracting one equation from the other we eliminate B and obtain

$$A=\frac{\mu\lambda^2-\mu'\lambda'^2}{\lambda^2-\lambda'^2}$$

But when $\lambda=\infty$, $\mu=A$. Hence

$$\mu_{\lambda=\infty}=\frac{\mu\lambda^2-\mu'\lambda'^2}{\lambda^2-\lambda'^2}$$

The following are the results of the experiments on glass :—

REFRACTIVE index of double extra-dense flint glass.

Specific gravity $\begin{cases} 4\cdot421 & \text{GORDON.} \\ 4\cdot42162 & \text{HOPKINSON.} \end{cases}$

$\sqrt{K}=1\cdot7783.$

Ray.	Wave length.	μ. GORDON.	μ. HOPKINSON.	Ray.	Wave length.
Bright line in extreme red	..	1·7032	1·7011	B	6·867
D	5·892	1·7099	1·7102	D	
b_1	5·174	1·7201	1·7209	b	
b_2		1·7234			
Bright line in central blue	..	1·7374	1·7272	F	
Band in extreme violet	..	1·7460	1·7432	G	
			1·7577	H_1	3·968

Which give $\mu_{\lambda=\infty}=\begin{cases} 1\cdot655 \text{ GORDON from D and } b. \\ 1\cdot672 \text{ HOPKINSON from B and } H_1. \end{cases}$

REFRACTIVE index of extra-dense flint glass.

Specific gravity $\begin{cases} 3\cdot884 & \text{GORDON.} \\ 3\cdot88947 & \text{HOPKINSON.} \end{cases}$

$\sqrt{K}=1\cdot7474.$

Ray.	Wave length.	μ. GORDON.	μ. HOPKINSON.	Ray.	Wave length.
Bright line in extreme red	..	1·6423	1·6429	B	6·867
D	5·892	1·6478	1·6505	D	
b_1	5·174	1·6561	1·6591	b	
b_2		1·6586			
Bright line in central blue	..	1·6693	1·6642	F	
Band in extreme violet	..	1·6757	1·6770	G	
			1·6885	H_1	3·968

Which give $\mu_{\lambda=\infty}=\begin{cases} 1\cdot615 \text{ GORDON from D and } b. \\ 1\cdot620 \text{ HOPKINSON from B and } H_1. \end{cases}$

REFRACTIVE index of light flint glass.

Specific gravity $\begin{cases} 3{\cdot}198 & \text{GORDON.} \\ 3{\cdot}20609 & \text{HOPKINSON.} \end{cases}$

$$\sqrt{K}=1{\cdot}7342.$$

Ray.	Wave length.	μ. GORDON.	μ. HOPKINSON.	Ray.	Wave length.
Bright line in extreme red	..	1·5685	1·5685	B	6·867
D.	5·892	1·5722	1·5740	D	
b_1.	5·174	1·5787	1·5803	b	
b_2.		1·5805			
Bright line in central blue	..	1·5879	1·5839	F	
Band in extreme violet	..	1·5920	1·5928	G	
			1·6007	H_1	3·968

Which give $\mu_{\lambda=\infty}=\begin{cases} 1{\cdot}547 \text{ GORDON from D and } b. \\ 1{\cdot}555 \text{ HOPKINSON from B and } H_1. \end{cases}$

REFRACTIVE index of hard crown glass.

Specific gravity $\begin{cases} 2{\cdot}484 & \text{GORDON.} \\ 2{\cdot}48664 & \text{HOPKINSON.} \end{cases}$

$$\sqrt{K}=1{\cdot}7629.$$

Ray.	Wave length.	μ. GORDON.	μ. HOPKINSON.	Ray.	Wave length.
Bright line in extreme red	..	1·5022	1·5141	B	6·867
D.	5·892	1·5045	1·5176	D	
b_1.	5·174	1·5083	1·5215	b	
b_2.		1·5093			
Bright line in central blue	..	1·5017	1·5236	F	
Band in extreme violet	..	1·5132	1·5288	G	
			1·5333	H_1	3·968

Which give $\mu_{\lambda=\infty}=\begin{cases} 1{\cdot}491 \text{ GORDON from D and } b. \\ 1{\cdot}504 \text{ HOPKINSON from B and } H_1. \end{cases}$

I here give for what it may be worth an attempt to determine for two glasses for what wave length of light μ would equal $\sqrt{K}$.

To determine the wave length of the ray for which $\mu=\sqrt{K}$, we have

$$\mu=A+\frac{B}{\lambda^2}$$

For double extra-dense flint we have, using HOPKINSON'S numbers,

$$A=1\cdot672.$$

For the D line the equation becomes

$$1\cdot710=1\cdot672+\frac{B}{3\cdot471}$$

or

$$B=3\cdot471(1\cdot710-1\cdot672)$$
$$=\cdot1319$$

Putting $\mu=\sqrt{K}=1\cdot778$, we have

$$\lambda=3\cdot527$$

In DRAPER'S photograph line N is 3·528

EXTRA-dense flint.

For the D line we have

$$1\cdot650=1\cdot620+\frac{B}{3\cdot471}$$

or

$$B=3\cdot471(\cdot030)=\cdot1041$$

Putting $\mu=\sqrt{K}=1\cdot747$, we have

$$1\cdot747=1\cdot620+\frac{\cdot1041}{\lambda^2}$$
$$\lambda=2\cdot862$$

The following table compares the various refractive indices of each dielectric with the square root of the specific inductive capacity:—

TABLE comparing μ and $\sqrt{K}$.

Dielectric.	$\sqrt{K}$.	$\mu_{\lambda=\infty}$.	μ_D.	μ_{H_1}.	$\lambda_{\mu=\sqrt{K}}$.
Double extra-dense flint glass	1·778	1·672	1·710	1·757	3·527, the wave length for N in ultra violet.
Extra-dense flint glass	1·747	1·620	1·650	1·688	2·862
Light flint glass*	1·734	1·555	1·574	1·601	
Hard crown glass	1·763	1·504	1·517	1·533	
Paraffin	1·4119	1·4220†			
Sulphur	1·606				
Bisulphide of carbon	1·345	..	1·611		
Common plate glass	1·801	..	1·543		

* In the abstract of the paper published in 'Proc. Roy. Soc.,' No. 191, the refractive indices for hard crown and light flint were interchanged. The mistake, which has only just been discovered, was caused by the polisher to whom the prisms were entrusted having interchanged the "H.C." and "L.F." labels.—June 17, 1879.

† GLADSTONE and CLERK MAXWELL, 'Maxwell's Electricity,' § 789, vol. ii., p. 389. The melting point of my paraffin was 68° C., that of Dr. GLADSTONE'S was less than 57° C.

Conclusion.

It will be seen from the foregoing table that the results of the experiments on certain dielectrics, notably two out of the four specimens of optical glass,* and the paraffin agree tolerably closely with the formula deduced from Professor CLERK MAXWELL's theory. For other dielectrics the formula does not even approximately hold good. I think we may fairly conclude that the square of the refractive index is a term in the expression of the specific inductive capacity, and that in some dielectrics it is by far the most important term. In other dielectrics it is overshadowed by some other term whose nature is at present unknown.

Pending the discovery of this unknown term, I propose the following method of eliminating it in a particular case :—

Let two plates of Iceland spar be constructed with their faces, perpendicular and parallel, to the axis of the crystal respectively. Let the specific inductive capacities be determined, and then if Professor CLERK MAXWELL's views of the nature of an electromagnetic disturbance in the ether are correct, I think we ought to have

$$\left\{\begin{matrix}\text{Ratio of specific inductive capacities}\\ \text{along and across axis.}\end{matrix}\right\}^{\frac{1}{2}}=\left\{\begin{matrix}\text{Ratio of refractive indices}\\ \text{along and across axis.}\end{matrix}\right\}$$

To me it seems possible that this equation may be unaffected by the existence of the unknown term, whatever it may be.†

I am proposing to continue this research, using 1000 LECLANCHÉ cells instead of secondary current; 500 of these are already erected, and a rapid reverser and chronograph have been constructed.

Since the appearance of Messrs. AYRTON and PERRY's paper on the viscosity of dielectrics,‡ it has become of interest to determine whether there is any change in the specific inductive capacity when the duration of the electrification is varied from $\frac{1}{500}$th second to several days.

It is impossible to use the coil for this work, as any variation in the speed of the break (or reverser) produces large changes in the electromotive force of secondary currents.

It is also important to determine whether difference of electromotive force produces any difference in specific inductive capacity. The agreement of my experiments on

* When newly cast. See note to page 438 (September 25, 1879).

† Note added June 17, 1879.—After many unsuccessful attempts to obtain plates of Iceland spar of the size required for my induction balance at anything like a reasonable price, I have at last, at the suggestion of Professor CORNU, decided to have a minature induction balance constructed, for the determination of the specific inductive capacity of spar and other expensive substances. This instrument is now almost finished; I hope a description of it, and of experiments performed with it, will form the subject of a future paper.

‡ 'Proc. Roy. Soc.,' 1877–8, vol. xxvii., p. 238.

paraffin with those of Messrs. GIBSON and BARCLAY, seems to indicate the non-existence of any change. The difference between my experiments and Dr. HOPKINSON'S may possibly, by some, be considered an indication that a change takes place. I hope a more or less complete investigation of this point will be the subject of a future memoir.*

I must not conclude this paper without expressing my great gratitude to Professor CLERK MAXWELL for his kindness in assisting me with advice and suggestions from time to time during the progress of the work. The original plan of the investigation, and the first sketches of the more important parts of the apparatus, are all due to him.

* Note added June 17, 1879.—Until now all attempts to use the battery instead of the coil have proved unsuccessful. No consistent results have been obtained with it. I think the want of success is due to the difficulty of discharging the apparatus, and of properly insulating it from the battery when the latter is not supposed to be connected to it. Four different commutators have already been tried without success. Professor CORNU has, however, had the kindness to devise another one for me, and this is now in course of construction; I have considerable hopes that it will answer its purpose.

September 25, 1879.—Some preliminary experiments made with the CORNU commutator, and a HOLTZ machine have been successful.